Bounding Volume Hierarchies

Def. BVH:

Given set S of objs, e.g. polygons.

If $|S| \leq k$: $BVH(S) =$ leaf node b, stores S;

else: $BVH(S) :=$ node v with n_v children $v_1, \ldots, v_n$,

where $v_i = BVH(S_i)$, $S_i \subseteq S$, $\dot{\bigcup} S_i = S$;

also v stores bounding volume $bv(v)$

(from a set of possible BV's),

s.t. $\forall p \in S: p \subseteq bv(v)$.

Types of BVH:

1. Layered BVH: $\forall$ children v_i: $bv(v_i) \subseteq bv(v)$
2. Wrapped BVH: $\forall$ leaves v_i under v: $S(v_i) \subseteq bv(v)$

Def.: "tightness"

Let S = surface / mesh, $B = \partial V$ s.t. $S \subseteq B$.

Def directed Hausdorff distance $h(B;S) = \max\limits_{p \in B} \min\limits_{q \in S} d(p,q)$

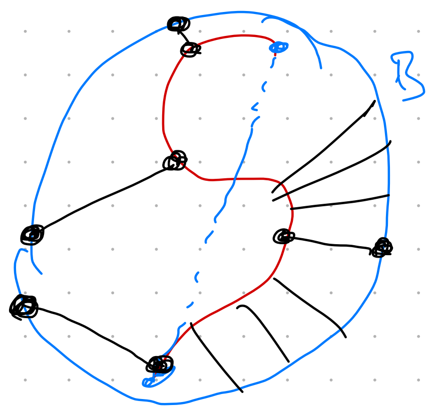
(usually $d = L_2$ norm)

(Note: h not symmetric!)

Def diameter $\operatorname{diam}(S) := \max\limits_{p,q \in S} d(p,q)$

Def tightness $$\tau(B;S) = \frac{h(B,S)}{\operatorname{diam}(S)}$$ "smaller is better"

Example:



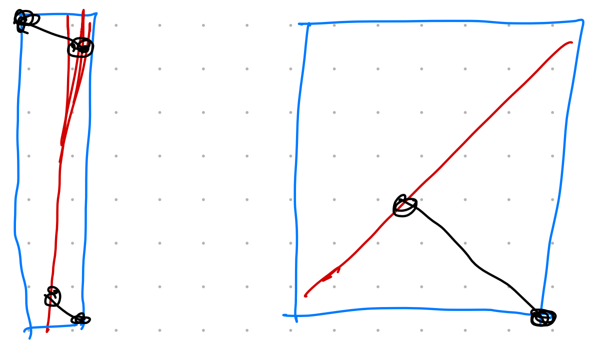
In case B = sphere,

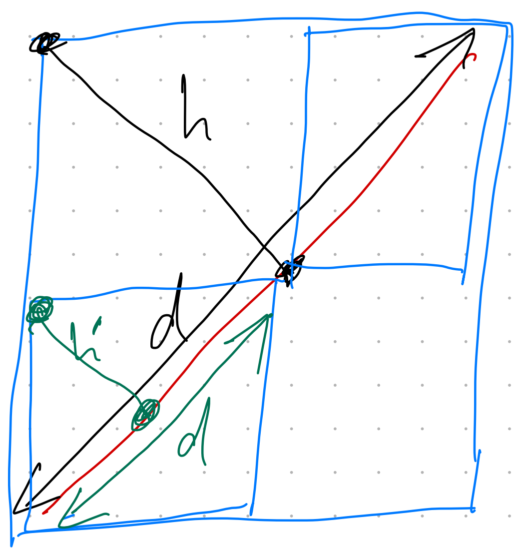
$0 < \tau \leq \frac{1}{2}$

Observations about tightness:

1. AABBs in a BVH:

Depends heavily on orientation of S

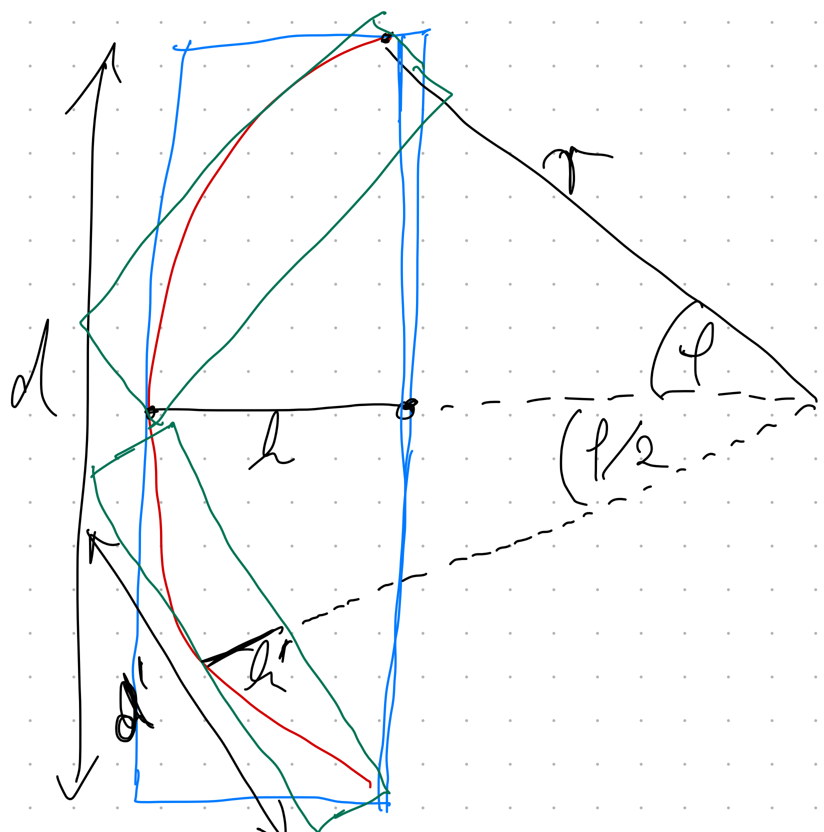




$$\tau = \frac{h}{d}$$

$$\tau' = \frac{h'}{d/2} = \frac{h/2}{d/2} = \tau \quad (!)$$

2. OBBs:



$$h = r(1 - \cos\varphi) \qquad d = 2r\sin\varphi$$

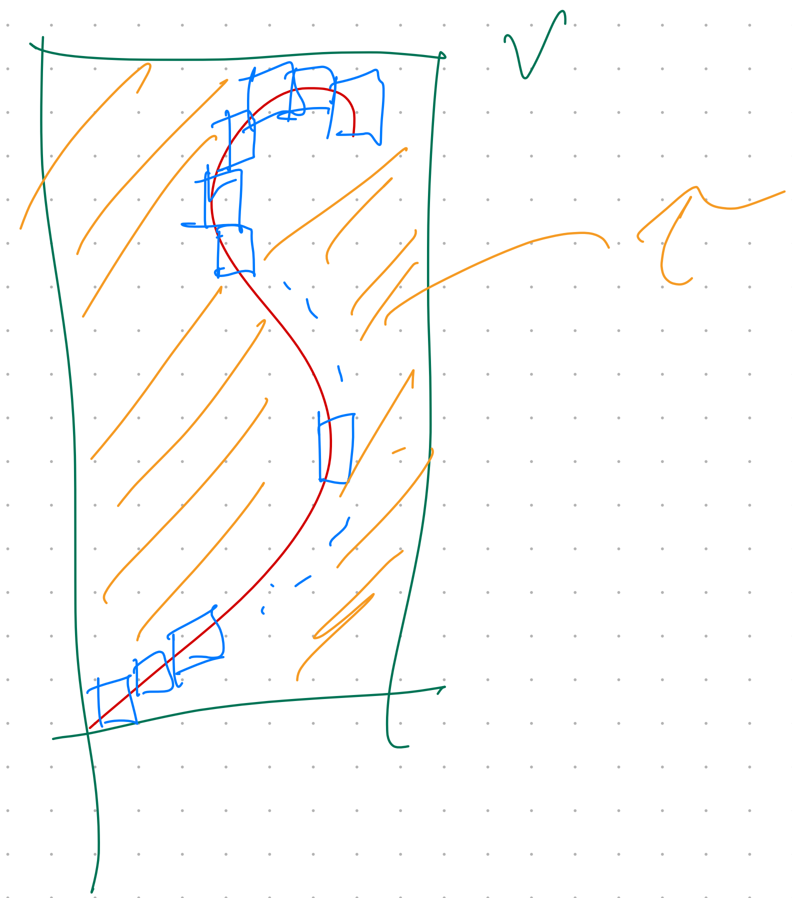
$$\tau = \frac{h}{d} = \frac{1-\cos\varphi}{2\sin\varphi} \underset{\text{Taylor}}{=} \frac{1-\left(1-\frac{1}{2!}\varphi^2 + \frac{1}{4!}\varphi^4 - \dots\right)}{2\left(\varphi - \frac{1}{3!}\varphi^3 + \dots\right)}$$

$$\approx \frac{\frac{1}{2}\varphi^2}{2\varphi} = \frac{1}{4}\varphi = O(\varphi)$$

$\tau \to 0$ as $\rho \to 0$

Another def. of tightness:

$$\tau := \frac{\mathrm{vol}(\, bv(v)\,)}{\sum_{p \in S(v)} \mathrm{vol}(\, bv(p)\,)}$$



BVH Construction

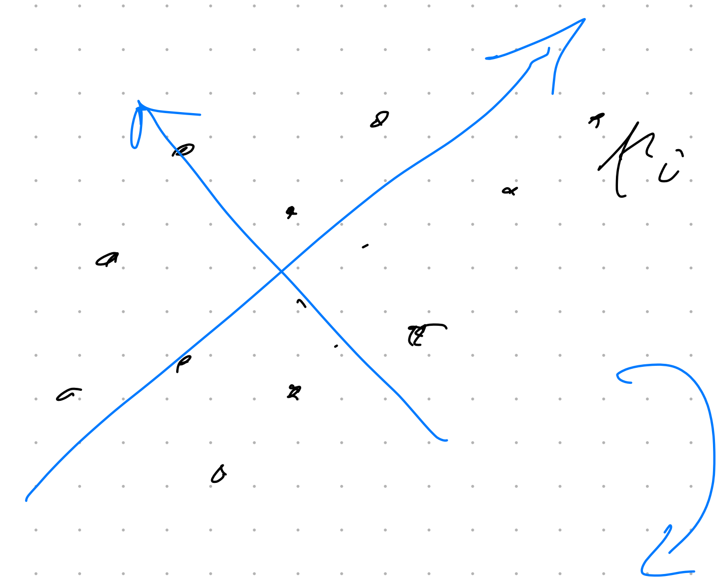
Given set S of pgons in 3D (set of elementary boxes)

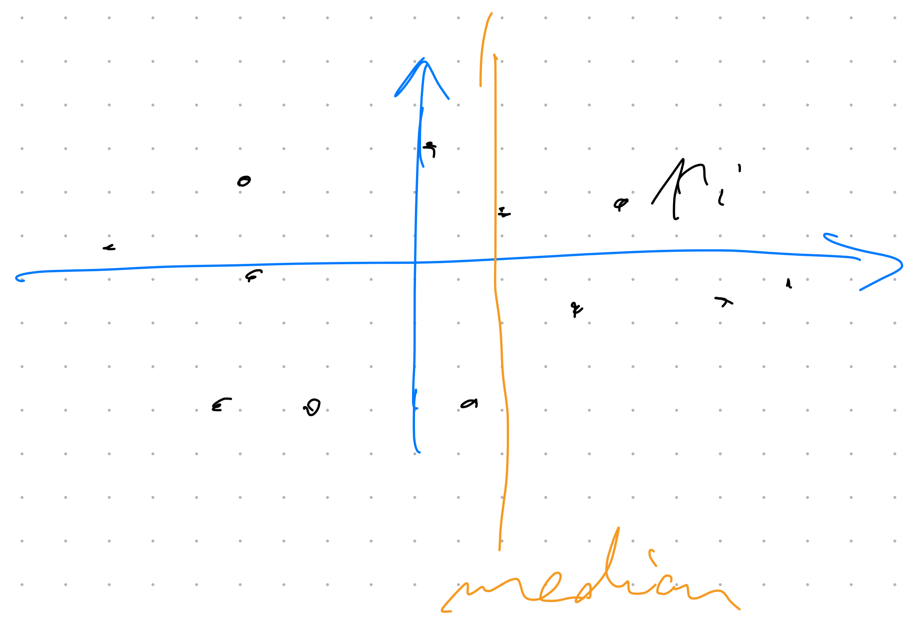
Consider the midpts $p_i \in S$

calculate PCA $\rightarrow$ transform $p_i \rightarrow p_i$

compute median along widest spread
(first PC)

$\rightarrow$ subsets L, R, $S = L \cup R$





Improvement: sweep-plane technique

Define cost fct:

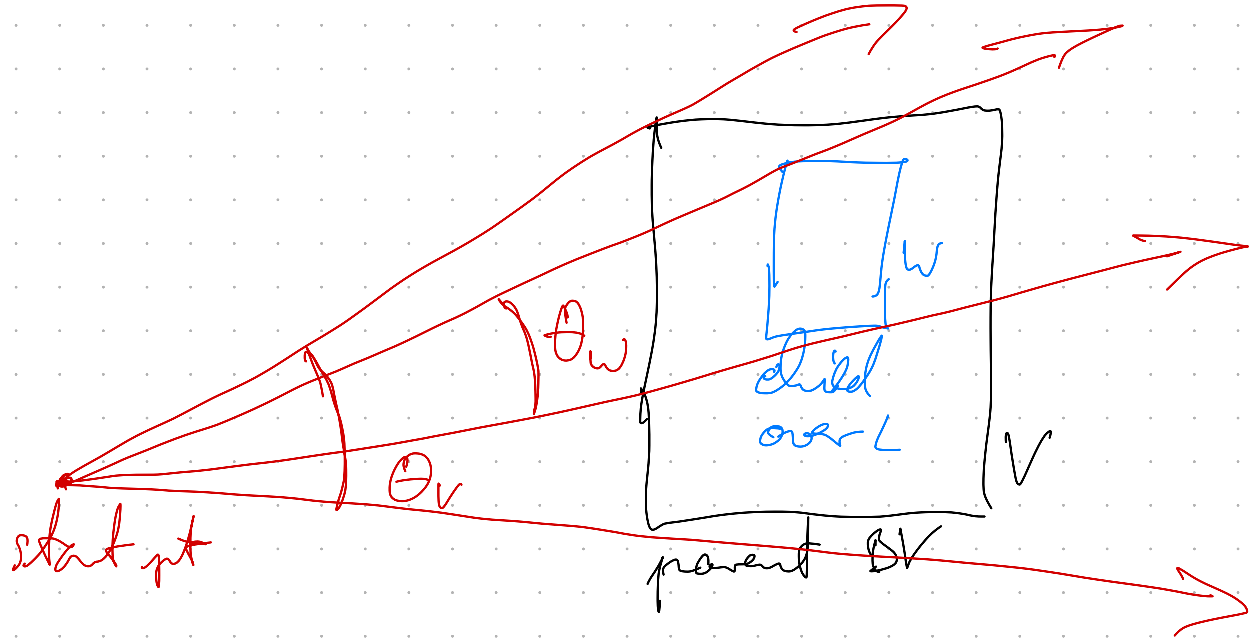
$$C(L,R) = \Pr[\text{traverse } L] \cdot C(L) + \Pr[\text{traverse } R] \cdot C(R)$$

Pr depends on application

Case ray tracing:

Pr[ray hits w | ray hits v]

$$= \frac{\Theta_w}{\Theta_v} \approx \frac{\text{area}(w)}{\text{area}(v)}$$



Simplified cost fct : Surface Area Heuristic (SAH)

$$c(L, R) = \frac{\text{area}(\text{bbox}(L))}{\text{area}(\text{bbox}(S))} \cdot |L| + \frac{\text{area}(\text{bbox}(R))}{\text{area}(\text{bbox}(S))} \cdot |R|$$

Wanted : $\min_{L,R} c(S)$ → sweep plane

sort all $p_i \in S$ → $p_1, p_2, \ldots, p_n$

for $j = 0, \ldots n$:
  calc cost $c(\{p_1, \ldots, p_j\}, \{p_{j+1}, \ldots, p_n\})$
  if smaller cost, than seen so far : remember j

In case of coll. det.:

$$\Pr[\text{traversal into } L] = \frac{\tau(\text{bbox}(L);\, L)}{\tau(\text{bbox}(S);\, S)}$$

Construction using Configuration Space:

S = set of elem. bboxes $b_i \subseteq \mathbb{R}^d$ $\quad (b_i = [a_i, b_i])$ (pts $\in \mathbb{R}^d$)

↳ consider as pts $p_i \in \mathbb{R}^{2d}$ $\quad (p_i = (a_i, b_i) \leftarrow$ one pt$)$

B = set of pts p_i

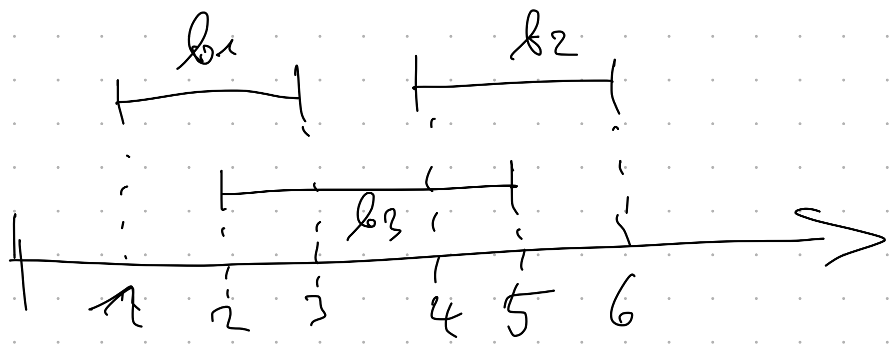
Build a kd-tree over B

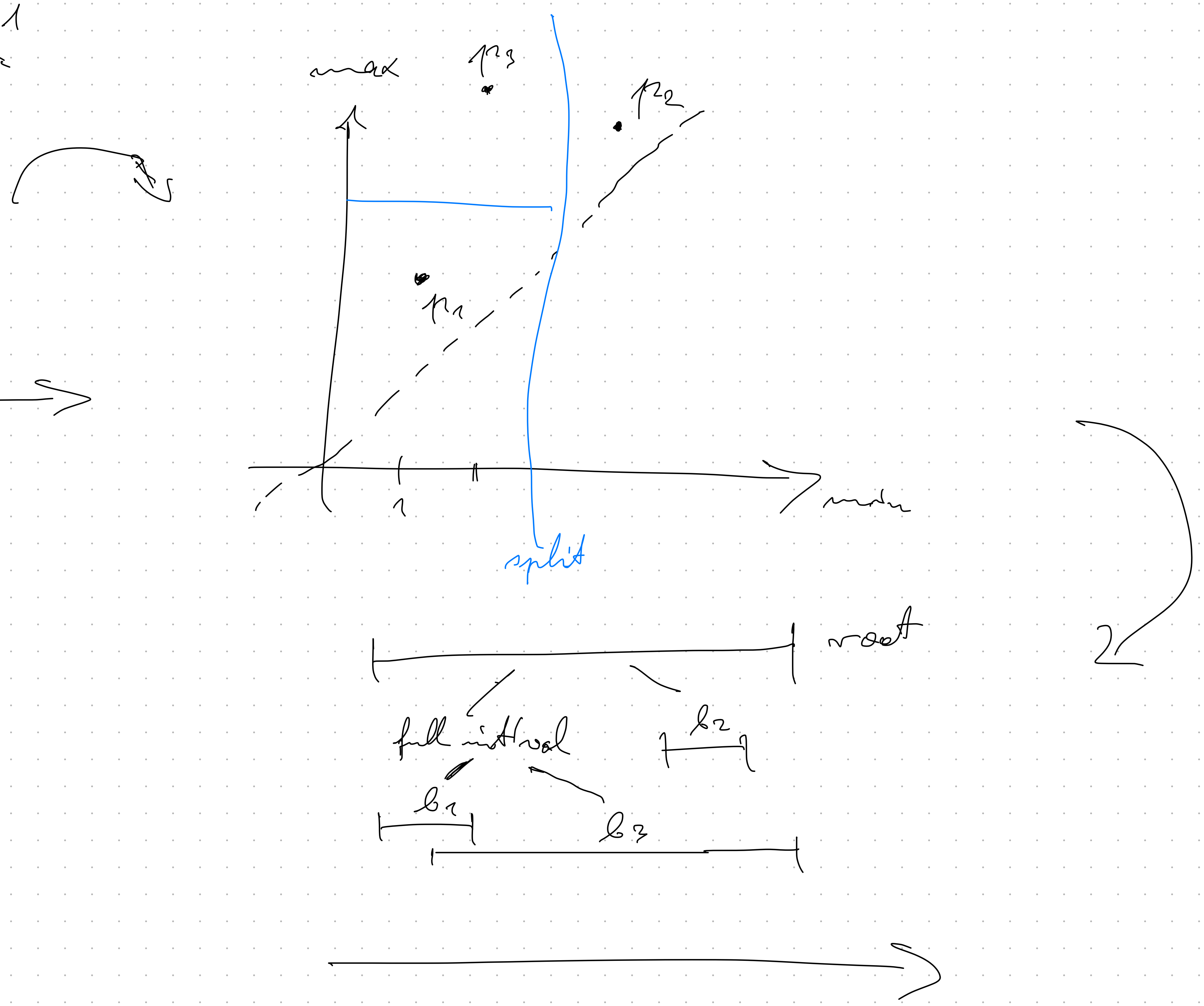
Transform the kd into a Box-Hierarchie as follows:

leaves in kd-tree $\Rightarrow$ leaves b_i in BVH inner nodes in kd-tree → inner nodes in BVH, v, with $bbox(v) := bbox(v_1, v_2)$

children

Example: in $\mathbb{R}^1$





Lemma: (w/o proof):

Rectangle intersection query in $\mathbb{R}^d$ can answered in time $O(n^{1-1/d} + k)$ in the worst case.

(with a suitable DVH, see above)

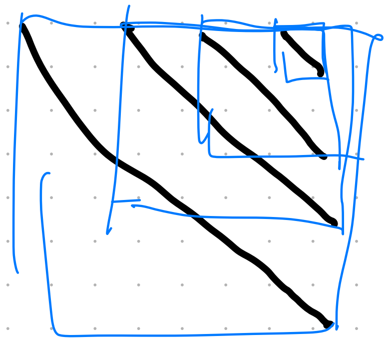
For $d \geq 3$, it can answered in time $\Omega(n^{1-1/d} + k)$

In 3D: $\Theta(n^{2/3})$

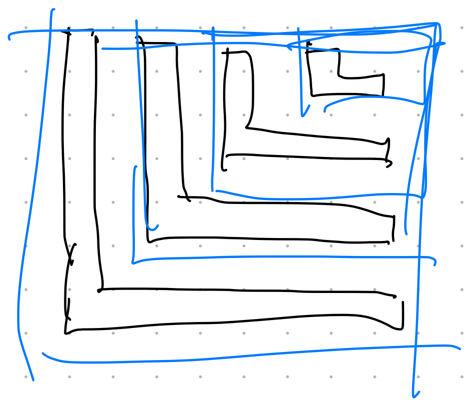
Analysis of Boxes for intersection tests:

Given n moving boxes (AABBs!)

Example:



$\rightsquigarrow$ $\Theta(n^2)$ intersecting pairs boxes!



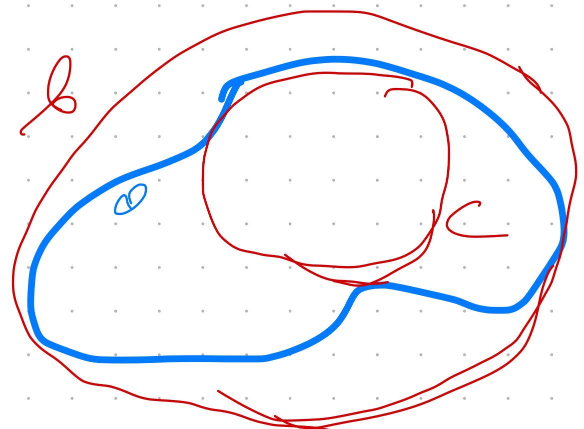
$\to$ n^2 overlaps!

Def.:

Let $S = \{o_1, \dots, o_n\}$ be set of objs with well-defined, non-empty volume

1. aspect ratio: For $o \in S$,

$$\alpha(o) := \frac{vol(b(o))}{vol(c(o))},$$



where $b(o)$ = smallest circumsphere

$c(o)$ = biggest in-sphere

2. $\alpha(S) := \min_{o_i \in S} \alpha(o_i)$

3. scale factor : $\sigma(S) := \min_{i,j} \frac{vol(b(o_i))}{vol(b(o_j))}$

Theorem (w/o proof):

Let S be set of objs in $\mathbb{R}^d$, $\alpha = \alpha(S)$, $\sigma = \sigma(S)$.

Let k_o = # pairs (o_i, o_j) overlap each other,

k_b = # pairs $(b(o_i), b(o_j))$

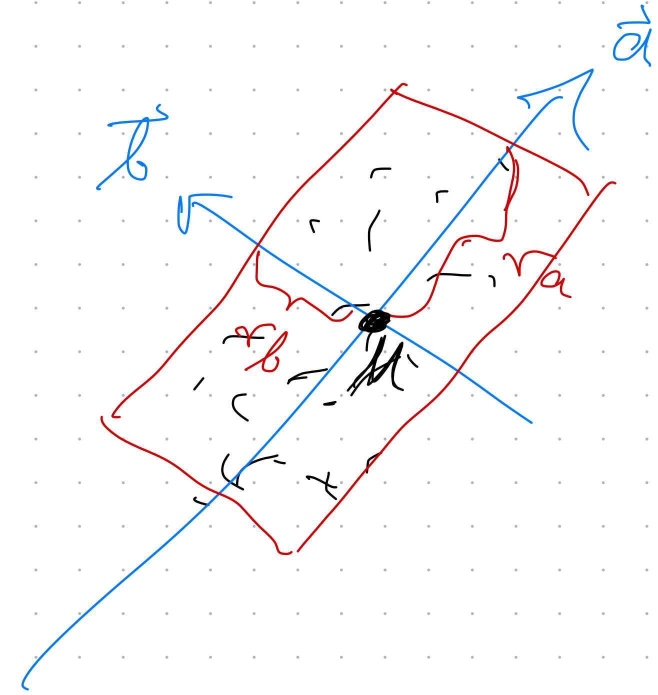
Then, $\frac{k_b}{n + k_o} \in O(\alpha \sqrt{\sigma} \log^2 \sigma)$.

if α, σ constant $\Rightarrow$ $k_b = O(k_o) + O(n)$

Constructing OBB:

Given set polygons p_i $\rightarrow$ midpts p_i

Do PCA trf $\rightarrow$ OBB extents,

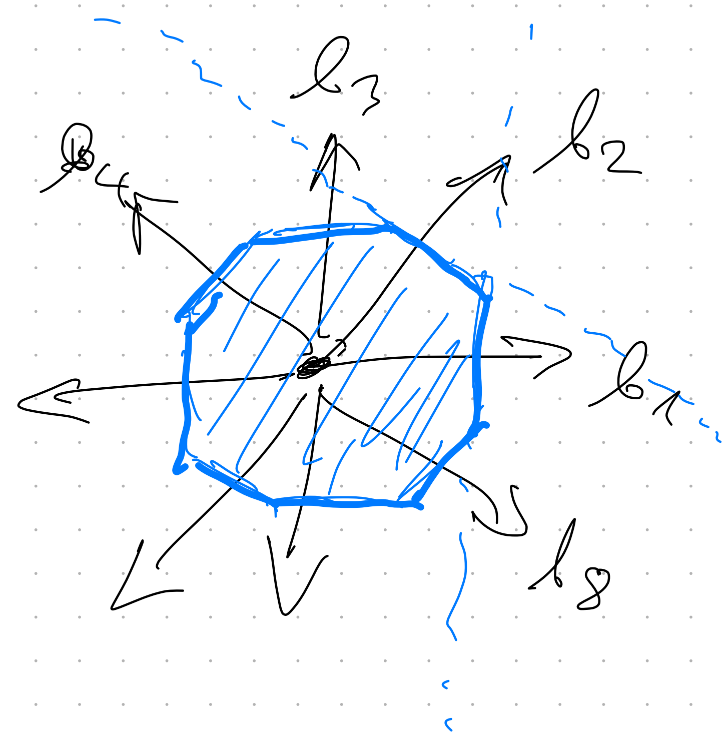


k-DOP:

Choose set of generators $B = \{\vec{b}_1, \ldots, \vec{b}_{2k}\}$
with $\vec{b}_{k+i} = -b_i$

B will be fixed

A k-DOP is $D = \bigcap_{i=1}^{2k} H_i$,



where H_i is half-space $\quad x \cdot \vec{b}_i - d_i \leq 0$

$\rightarrow \quad D = (d_1, \ldots, d_{2k}) \in \mathbb{R}^{2k}$

overlap test: